1) Air enters one-inlet, one-exit control volume at 8 bar, 600 K and 40 m/s through a flow area of 20 cm$^2$. At the exit, the pressure is 2 bar, the temperature is 400 K and the velocity is 350 m/s. The air behaves as ideal gas. For steady-state operation, determine:
   a) mass flow rate, in kg/s,
   b) the exit flow area, in cm$^2$

**KNOWN:** Air flows through a one-inlet, one-exit control volume. Data are known at the inlet and exit.

**FIND:** Determine (a) the mass flow rate, (b) the exit area.

**SCHEMATIC & GIVEN DATA:**

- $P_1 = 8$ bar
- $T_1 = 600$ K
- $V_1 = 40$ m/s
- $A_1 = 20$ cm$^2 = 0.002$ m$^2$
- $P_2 = 2$ bar
- $T_2 = 400$ K
- $V_2 = 350$ m/s

**ASSUMPTIONS:** (1) The control volume is at steady state. (2) The air behaves as an ideal gas. (3) The flow is one-dimensional at the inlet and exit.

**ANALYSIS:** Beginning with the mass rate balance

$$\frac{dm}{dt} = m_1 - m_2$$

$$\Rightarrow m_1 = m_2 = m$$

Using data at the inlet and the ideal gas equation of state

(a) $$m = \rho A_1 V_1 = \left(\frac{P}{RT_1}\right) A_1 V_1$$

$$= \frac{(8 \text{ bar})}{(8.314 \text{ kJ/mol K}) (600 \text{ K})} \frac{10^5 \text{ N m}^2}{1 \text{ bar}} \frac{1 \text{ kJ}}{10^3 \text{ N m}} \left(0.002 \text{ m}^2\right)(40 \text{ m/s})$$

$$= 0.3717 \text{ kg/s}$$
(b) From $\dot{m}_1 = \dot{m}_2$

\[ S_1 A_1 V_1 = S_2 A_2 V_2 \]

Thus

\[ A_2 = \left( \frac{S_1}{S_2} \right) \left( \frac{V_1}{V_2} \right) A_1 \]

With $\phi = \frac{P}{RT}$

\[ A_2 = \left( \frac{P_2}{P_1} \right) \left( \frac{T_1}{T_2} \right) \left( \frac{V_1}{V_2} \right) A_1 \]

\[ = \left( \frac{\phi}{2} \right) \left( \frac{400}{600} \right) \left( \frac{40}{350} \right) (20 \text{ cm}^2) \]

\[ = 0.095 \text{ cm}^2 \]

$A_2$
2) R134a at 700 kPa and 100 °C enters an adiabatic nozzle with a velocity of 20 m/s and leaves at 320 kPa and 30 °C. Find the exit velocity and the ratio of the inlet to exit areas.

\[ \dot{m} \Delta h + \Delta \text{ke} + \Delta \dot{\text{pe}} = 0 \]

Since \( \dot{m} \neq 0 \), then \( \Delta h + \Delta \text{ke} + \Delta \dot{\text{pe}} = 0 \)

So \( \Delta h = \Delta \text{ke} \)

From Table A-12:

\begin{align*}
\text{At} & \quad p_1 = 0.7 \text{ MPa} \\
\text{and} & \quad T_1 = 100 \, ^\circ \text{C} \\
\text{we have} & \quad h_1 = 338.19 \text{ kJ/kg}
\end{align*}

\begin{align*}
\text{At} & \quad p_2 = 0.32 \text{ MPa} \\
\text{and} & \quad T_2 = 30 \, ^\circ \text{C} \\
\text{we have} & \quad h_2 = 274.28 \text{ kJ/kg}
\end{align*}
\[ \Delta h = \frac{63.91 \text{ kgs}}{\text{kg}} = \Delta \text{ke} \]

\[ \Delta \text{ke} = \frac{V_2^2 - V_1^2}{2} = \frac{63.91 \text{ kgs}}{\text{kg}} \]

\[ V_2^2 = 2 \left( \frac{63.91 \text{ kgs}}{\text{kg}} \right) + V_1^2 = 2 \left( \frac{63.91 \text{ kgs}}{\text{kg}} \right) + (20 \text{ m/s})^2 \]

Recall that \( 1 \text{ kgs} = 1000 \text{f} = 1000 \text{N.m} \Rightarrow m = 1000 \text{ kgs m}^2/\text{s}^2 \)

\[ V_2^2 = 2 \left( \frac{63.91 \text{ kgs}}{\text{kg}} \times 1000 \right)m^2/\text{s}^2 + (20)^2 m^2/\text{s}^2 \]

\[ V_2 = (127800 + 400)^{m^2/\text{s}^2} \]

\[ V_2 = 358 \text{ m/s} \]

\[ \dot{m} = \text{const} \Rightarrow \rho V A_1 = \rho_2 V_2 A_2 \]

\[ \frac{A_1}{A_2} = \frac{\rho_2 V_2}{\rho_1 V_1} = \frac{\nu_1 V_1}{\nu_2 V_2} \]

From Table A.12

\[ \frac{\nu_1}{\nu_2} = 0.04064 \text{ m}^3/\text{kg} \]

\[ \frac{\nu_1}{\nu_2} = 0.07214 \text{ m}^3/\text{kg} \]

\[ \frac{A_1}{A_2} = \frac{(0.04064)(358)}{(0.07214)(20)} = 10.1 \]
3) Air enters an insulated diffuser operating at steady state with a pressure of 1 bar, a temperature of 300 K and a velocity of 250 m/s. At the exit, the pressure is 1.13 bar and the velocity is 140 m/s. Potential energy effects can be neglected. Using the ideal gas model,

a) the ratio of exit flow area to the inlet flow area
b) the exit temperature in K

Known data is provided for a diffuser at steady state, through which air is flowing.

Find: Determine the ratio of the exit flow area to the inlet flow area, and the exit temperature.

**Schematic & Given Data:**

\[
\begin{align*}
A_1 & = & \text{Inlet Flow Area} \\
A_2 & = & \text{Exit Flow Area} \\
P_1 & = & 1 \text{ bar} \\
P_2 & = & 1.13 \text{ bar} \\
T_1 & = & 300 \text{ K} \\
T_2 & = & \text{Exit Temperature} \\
V_1 & = & 250 \text{ m/s} \\
V_2 & = & 140 \text{ m/s} \\
\end{align*}
\]

**Assumptions:**
1. The control volume shown in the schematic is at steady state.
2. For the control volume, \( \dot{Q}_{cv} = \dot{W}_{cv} = 0 \), and potential energy effects can be ignored. 3. Air is modeled as an ideal gas with constant \( c_p \).

**Analysis:**

The mass rate balance reads \( m_2 = m_1 \), or

\[
\frac{A_2 V_2}{P_2} = \frac{A_1 V_1}{P_1} \Rightarrow \frac{A_2}{A_1} = \left( \frac{P_1}{P_2} \right) \left( \frac{V_1}{V_2} \right) \tag{1}
\]

The exit temperature, \( T_2 \), can be obtained using an energy rate balance:

\[
0 = \dot{Q}_{cv} - \dot{W}_{cv} + m \left[ h_2 - h_1 + \frac{V_1^2 - V_2^2}{2} + g(3 \times 2.1) \right]
\]

\[
h_2 = h_1 + \frac{V_1^2}{2} - \frac{V_2^2}{2}
\]

Using data from Table A-22:

\[
h_2 = (300.19 \text{ kJ/kg}) + \left( \frac{250^2 - 140^2}{2} \right) \text{ kJ/kg m/s}^2 \times 1 \text{ Nm/1000 Nm} = 321.64 \text{ kJ/kg}
\]

Then, interpolating in Table A-22 for \( h_2 = 321.64 \text{ kJ/kg} \)

\[T_2 = 321.3 \text{ K} \]

Returning to Eq. (1):

\[
\frac{A_2}{A_1} = \left( \frac{P_2}{P_1} \right) \left( \frac{V_1}{V_2} \right) = 1.692
\]

\( \frac{A_2}{A_1} \)
4) Air is to be compressed from 120 kPa and 310 K to 700 kPa and 430 K. A heat loss of 20 kJ/kg occurs during the compression. Δke=0. The mass flow rate is 90 kg/min. Find the required power input.

\[
\dot{W} = Q - \dot{W} = \dot{m}(h_2 - h_1)
\]

\[
\dot{W} = Q - \dot{m}(h_2 - h_1) = Q - \dot{m} C_p (T_2 - T_1)
\]

\[
\dot{W} = Q + \dot{m} C_p (T_2 - T_1)
\]

\[
\dot{W} = -20 \text{ kJ/kg} \times \left(90 \frac{\text{kg}}{\text{min}}\right) - \left(90 \frac{\text{kg}}{\text{min}}\right) \times 1.008 \text{ kJ/kg k} \times (430 - 310) \text{k}
\]

\[
\Rightarrow \dot{W} = -12686 \frac{\text{kJ}}{\text{min}} \left(\frac{1\text{ min}}{60\text{ s}}\right) \approx -211\text{ kW}
\]
5) Air expands through a turbine from 10 bar, 900 K to 1 bar, 500 K. The inlet velocity is small compared to exit velocity of 100 m/s. The turbine operates at steady-state and develops an output of 3200 kW. Heat transfer between the turbine and its surroundings and potential energy effects are negligible. Calculate the mass flow rate of air in kg/s and the exit area in m².

**KNOWN:** Air expands through a turbine with known conditions at the inlet and exit. The power developed is known.

**FIND:** Determine the mass flow rate and the exit area.

**SCHEMATIC & GIVEN DATA:**

\[ P_1 = 10 \text{ bar} \quad T_1 = 900 \text{ K} \quad V_1 \ll V_2 \]

\[ P_2 = 1 \text{ bar} \quad T_2 = 500 \text{ K} \quad V_2 = 100 \text{ m/s} \]

\[ W_{cv} = 3200 \text{ kW} \]

**ASSUMPTIONS:**

1. The control volume is at steady state.
2. Heat transfer is negligible.
3. Potential energy effects and kinetic energy at the inlet can be neglected.
4. The air behaves as an ideal gas.

**ANALYSIS:** Begin with a steady-state energy balance

\[ 0 = \dot{Q}_{cv} + W_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{V_2^2}{2} \right] + Q \]

where \( \dot{m}_1 = \dot{m}_2 = \dot{m} \). Solving for \( \dot{m} \)

\[ \dot{m} = \frac{W_{cv}}{(h_1 - h_2) - \frac{V_2^2}{2}} \]

From Table A-22; \( h_1 = 932.93 \text{ kJ/kg} \) and \( h_2 = 503.02 \text{ kJ/kg} \). Thus

\[ \dot{m} = \frac{(3200 \text{ kW}) \left(1 \text{ kJ/s/1 kW}\right)}{(932.93 - 503.02) \text{ kJ/kg} - \left(100^2 \text{ m}^2/\text{s}^2\right)} = \frac{1 \text{ N} \cdot \text{m/s}^2}{\text{kJ/m}^2/\text{m}^2} \cdot\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \]

\[ \dot{m} = 7.53 \text{ kg/s} \]

The exit area is

\[ A_2 = \frac{V_2 \dot{m}}{P_2} = \frac{RT_2 \dot{m}}{P_2 V_2} \]

\[ = \frac{(8.314 \text{ kJ/mol K})(500 \text{ K})(7.53 \text{ kg/s})}{(1 \text{ bar})(100 \text{ m/s})} \cdot\frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \cdot\frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \]

\[ A_2 = 0.108 \text{ m}^2 \]

1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.
Cogeneration is often used where a steam supply is needed for industrial process energy. Assume a supply of 5 kg/s steam at 0.5 MPa is needed. Rather than generating this from a pump and boiler, the setup in figure is used to extract the supply from the high-pressure turbine. Find the power the turbine now cogenerates in this process.

![Diagram of steam flow process]

C.V. Turbine, steady state, 1 inlet and 2 exit flows, assume adiabatic, \( \dot{Q}_C = 0 \):

Continuity Eq. 6.9: \( \dot{m}_1 = \dot{m}_2 + \dot{m}_3 \)

Energy Eq. 6.10: \( \dot{Q}_C + \dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}_T \);

Supply state 1: 20 kg/s at 10 MPa, 500°C
Process steam 2: 5 kg/s, 0.5 MPa, 155°C,
Exit state 3: 20 kPa, \( x = 0.9 \)
Table B.1.3: \( h_1 = 3373.7 \), \( h_2 = 2755.9 \) kJ/kg,
Table B.1.2: \( h_3 = 251.4 + 0.9 \times 2358.3 \)
\[ = 2373.9 \text{ kJ/kg} \]

\[ \dot{W}_T = 20 \times 3373.7 - 5 \times 2755.9 - 15 \times 2373.9 = 18.084 \text{ MW} \]
The compressor in a plant (see figure) receives carbon dioxide at 100 kPa, 280 K, with a low velocity. At the compressor discharge, the carbon dioxide exits at 1100 kPa, 500 K, with velocity of 25 m/s and then flows into a constant-pressure aftercooler (heat exchanger) where it is cooled down to 350 K. The power input to the compressor is 50 kW. Determine the heat transfer rate in the aftercooler.

\[ \dot{Q}_{\text{cool}} = (h_2 - h_1) + \frac{1}{2} (V_2^2 - V_1^2) + g \left( h_2 - h_1 \right) \]

\[ -\dot{W} = (h_2 - h_1) + \frac{1}{2} V_2^2 = 203.8 \text{ kJ/kg} \]

\[ \dot{m} = \frac{V_2}{\dot{W}} = \frac{-50}{-203.8} = 0.245 \text{ kg/s} \]

**CV for cooler**

\[ q - W = h_3 - h_2 + \frac{1}{2} (V_3^2 - V_2^2) + g \left( h_3 - h_2 \right) \]

\[ = h_3 - h_2 = -143.6 \text{ kJ/kg} \]

\[ \dot{Q}_{\text{c}} = -\dot{m} = 0.245 \times (-143.6) = 35.2 \text{ kW} \]
8) Separate streams of steam and air flow through the turbine and heat exchanger arrangement shown in Fig.P4.108. Steady-state operating data are provided on the figure. Heat transfer with the surroundings can be neglected, as can all kinetic and potential energy effects. Determine:

(a) \( T_3 \), in K,

(b) the power output of the second turbine, in kW.

\[ T_1 = 600^\circ C, \quad \rho_1 = 20 \text{ bar} \]

\[ T_2 = 400^\circ C, \quad \rho_2 = 10 \text{ bar} \]

\[ T_3 = ? \]

\[ T_4 = 240^\circ C, \quad \rho_4 = 1 \text{ bar} \]

\[ T_6 = 1200 \text{ K}, \quad \rho_6 = 1 \text{ bar} \]

\[ m_6 = 1500 \text{ kg/min} \]
ANALYSIS:

(a) To determine the steam mass flow rate, write an energy rate balance for turbine 1 and use data from Table A-2.

$$ \dot{m}_1 = \frac{\dot{W}_{11}}{h_1 - h_2} \Rightarrow \dot{m}_1 = \frac{\dot{W}_{11}}{h_1 - h_2} = \frac{10,000 \text{ kW}}{(1293.1 - 3263.9) \text{kJ/kg}} $$

$$ = 28.46 \text{ kg/s} $$

Next, an energy rate balance for the heat exchanger reduces to

$$ 0 = \dot{m}_2 \left[ h_2 - h_3 \right] + \dot{m}_3 \left[ h_3 - h_6 \right] $$

With data from Table A-2.

$$ h_3 = h_2 + \frac{\dot{m}_3}{m_2} \left[ h_3 - h_6 \right] = 3263.9 \frac{\text{kJ}}{\text{kg}} + \frac{(550.26) \text{kJ}}{23.46 \text{kg/s}} \left[ (1258.9 - 1127.6) \frac{\text{kJ}}{\text{kg}} \right] $$

$$ = 3645.6 \frac{\text{kJ}}{\text{kg}} $$

Interpolating in Table A-4 at 10 bar gives $T_3 = 576 \text{ C}$.

(b) An energy rate balance for turbine 2 is

$$ \dot{W}_{21} = \dot{m}_3 \left( h_3 - h_4 \right) = 23.46 \frac{\text{kg/s}}{} \left[ 3645.6 - 2775.2 \right] \frac{\text{kJ}}{\text{kg}} \left| \frac{4 \text{ kW}}{1 \text{ kJ/s}} \right| $$

$$ = 16,215 \text{ kW} $$
9) A pump steadily delivers water through a hose terminated by nozzle. The exit of the nozzle has a diameter of 2.5 cm and is located 4 m above the pump inlet pipe, which has a diameter of 5 cm. The pressure is equal to 1 bar at both the inlet and the exit, and the temperature is constant at 20°C. The magnitude of the power input required by the pump is 8.6 kW and the acceleration of gravity is \( g = 9.81 \text{ m/s}^2 \). Determine the mass flow rate delivered by the pump in kg/s.

**KNOWN:** Water flows through pumping system with known inlet and exit conditions. The power required by the pump is also specified.

**FIND:** Determine the mass flow rate.

**SCHEMATIC & GIVEN DATA:**

**ASSUMPTIONS:** (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) The water behaves as an incompressible liquid. (4) The acceleration of gravity is constant at \( g = 9.81 \text{ m/s}^2 \). (5) The temperature and pressure are nearly constant throughout.

**ANALYSIS:** To find \( \dot{m} \), begin with steady-state mass and energy rate balances

\[
0 = \dot{Q} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]
\]

where \( \dot{m} = \dot{m}_1 = \dot{m}_2 = \dot{m} \), and the specific enthalpy term is eliminated based on assumption (3) and Eq. 5.20b.

From Eq. 5.3a, \( V = \dot{m} \frac{V}{\rho} \), and (*) becomes

\[
0 = -\dot{W}_{cv} + \dot{m} \left[ \frac{\dot{m} V/A_1}{2} \right] + \frac{m^3}{2} \frac{V_2^2}{A_2} \left( \frac{1}{A_2^2} - \frac{1}{A_1^2} \right) + \dot{m} g (z_1 - z_2)
\]

From \( A = \pi d^2 / 4 \); \( A_1 = 0.000991 \text{ m}^2 \) and \( A_2 = 0.001964 \text{ m}^2 \). Now, with \( V = V_{F, 20^\circ C} = 1.0018 \times 10^{-3} \text{ m}^3/\text{kg} \) from Table A-2, we can insert values into (*)

\[
0 = -(-8.6 \text{ kW}) \left[ \frac{1 \text{ kJ/s}}{1 \text{ kW}} + \left( \frac{1.0018 \times 10^{-3} \text{ m}^3}{\text{kg}} \right)^2 \frac{1}{2} \left( \frac{1}{0.000991^2} - \frac{1}{0.001964^2} \right) \frac{1}{\text{m}^4} \right]
\]

\[
0 = \left( \frac{1}{1 \text{ kg/m}^3} \right) \left( \frac{1 \text{ kJ}}{10^5 \text{ N m}} \right) + \frac{m^3}{10^9 \text{ N m}} (-4 \text{ m}) + \frac{1}{1 \text{ kg/m}^3} \left( \frac{1 \text{ kJ}}{10^5 \text{ N m}} \right)
\]

or

\[
0 = 8.6 - 1.9513 \times 10^{-3} \text{ m}^3 - 0.03924 \dot{m} \text{ (where } \dot{m} \text{ is in kg/s)}
\]

This equation is cubic in \( \dot{m} \). The solution is

\[
\dot{m} = 15.98 \text{ kg/s}
\]
10) R-134-a is throttled from 800 kPa and 25°C to a final temperature of -20 °C. Find the pressure and internal energy of the refrigerant at the final state.

From Table A-10:

\[
\begin{align*}
\text{State 1} &: \quad h_{\text{sat}} = 8.12 \text{ J/kg} \\
\text{State 2} &: \quad h_{\text{g}, -20^\circ\text{C}} = 24.26 \text{ kJ/kg} \\
\quad &\quad h_{\text{g}, -20^\circ\text{C}} = 23.31 \text{ kJ/kg} \\
\quad &\quad h_{\text{g}, -20^\circ\text{C}} = 21.05 \text{ kJ/kg}
\end{align*}
\]

\[h_2 = h_1 = 84.32 \text{ kJ/kg} \rightarrow \text{Wet Mixture}\]

\[\rho = \rho_{\text{sat}, -20^\circ\text{C}} = 0.13299 \text{ m³/kg} = 1.3299 \text{ bar}\]

Find the quality:
\[h = h_f + x(h_{fg})\]
\[84.32 = 24.26 + x(21.05)\]
\[x = 0.284\]

\[\nu = \nu_f + x \nu_g\]
\[\nu = 24.17 + 0.284(191.67)\]
\[\nu = 78.60 \text{ kJ/kg}\]
11) Ammonia enters a heat exchanger operating at steady-state as superheated vapor at 14 bar, 60°C, where it is cooled and condensed to saturated liquid at 14 bar. The mass flow rate of the refrigerant is 450 kg/h. A separate stream of air enters the heat exchanger at 17°C, 1 bar and exits at 42°C, 1 bar. Ignoring heat transfer from the outside of the heat exchanger and neglecting kinetic and potential energy effects, determine mass flow rate of the air, in kg/min.

**Known:** Ammonia and air pass in separate streams through a heat exchanger at steady state, for which data are provided.

**Find:** Determine the mass flow rate of the air.

**Schematic & Given Data:**

- **NH₃**
  - Sup. vapor
  - 14 bar, 60°C
  - Mass flow rate: 450 kg/h

- **Air**
  - 17°C, 1 bar
  - Exit: 42°C, 1 bar

**Assumptions:**
1. A control volume enclosing the heat exchanger is at steady state.
2. For the control volume, \( \dot{W}_{vap} = 0 \), heat transfer can be ignored, and kinetic/potential energy effects are negligible.
3. Air is modeled as an ideal gas.

**Analysis:** Since the streams flow separately, the conservation of mass principle indicates at steady state: \( \dot{m}_1 = \dot{m}_3 = \dot{m}_R \) and \( \dot{m}_2 = \dot{m}_A \).

An energy rate balance reads:

\[
0 = \dot{m}_R [h_2 - h_1] + \dot{m}_R \left[ \frac{h_2 - h_3}{\gamma - 1} + \frac{q}{\gamma - 1} \right] - \dot{m}_A \left[ h_3 - h_4 \right]
\]

\[
\Rightarrow \quad \dot{m}_A = \frac{\dot{m}_R [h_2 - h_1]}{h_4 - h_3}
\]

From Table A-15, \( h_1 = 1542.88 \text{kJ/kg} \). From Table A-14, \( h_2 = 352.97 \text{kJ/kg} \). From Table A-22, \( h_3 = 290.16 \text{kJ/kg} \), \( h_4 = 315.27 \text{kJ/kg} \). Then

\[
\dot{m}_A = \frac{(450 \text{ kg}) [1542.89 - 352.97]}{315.27 - 290.16} = 355.4 \text{ kg/min}
\]

1. The validity of the ideal gas model is readily checked using the generalized compressibility chart.
The cooling coil of an air-conditioning system is a heat exchanger in which air passes over tubes through which R-22 flows. Air enters with a volumetric flow rate of 40 m³/min at 27°C, 1.1 bar, and exits at 15°C, 1 bar. Refrigerant enters the tubes at 7 bar with a quality of 16% and exits at 7 bar, 15°C. Ignoring heat transfer from the outside of the heat exchanger and neglecting kinetic and potential energy effects, determine at steady-state:

a) the mass flow rate of refrigerant, in kg/min
b) the rate of energy transfer, in kJ/min, from the air to the refrigerant

**Known:** Air and Refrigerant 22 pass in separate streams through a heat exchanger. Data are known at the inlet and exit of each stream.

**Find:** Determine (a) the mass flow rate of refrigerant and (b) the rate of energy transfer from the air to the refrigerant.

**Schematic & Given Data:**

![Heat Exchanger Diagram]

**Assumptions**:
1. The control volume is at steady state. 02. Heat transfer from the outside of the heat exchanger is negligible, and Wcv = 0.
3. Kinetic and potential effects can be neglected. 34. The air behaves as an ideal gas, as can be verified by reference to the compressibility chart.

**Analysis:**

(a) The mass flow rate of refrigerant is determined using steady-state mass and energy balances. First, since the air and refrigerant flow as separate streams

\[ m_1 = \dot{m}_2 = \dot{m}_{\text{air}} \]

\[ m_3 = \dot{m}_4 = \dot{m}_{\text{R-22}} \]

Thus, the energy rate balance reduces as follows

\[ 0 = c_v \dot{m}_{\text{air}} \left[ h_1 - h_2 + \frac{\dot{V}^2}{2} + \frac{\dot{h}^2}{2} + g (z_1 - z_2) \right] + \dot{m}_{\text{R-22}} \left[ h_3 - h_4 + \frac{\dot{V}^2}{2} + g (z_3 - z_4) \right] \]

and

\[ \dot{m}_{\text{R-22}} = \dot{m}_{\text{air}} \left( \frac{h_1 - h_3}{h_4 - h_3} \right) \]

The mass flow rate of air is found using data at the inlet and the ideal gas equation of state

\[ \dot{m}_{\text{air}} = \frac{(AV)}{\rho_1} = \frac{P_1 (AV)}{RT_1} = \frac{(1.1 \text{ bar})(40 \text{ m}^3/\text{min})}{(0.314 \cdot \frac{28.97 \text{ kg/m}}{280 \text{ K})}(300 \text{ K}) = \frac{1 \text{ kJ}}{1 \text{ bar}} \frac{10^5 \text{ N/m}^2}{10^5 \text{ N-m}} \]

\[ = 51.11 \text{ kg/min} \]
From Table A-22; \( h_1 = 300.19 \text{ kJ/kg} \) and \( h_2 = 288.15 \text{ kJ/kg} \). Further, using data from Table A-8,

\[
 h_3 = h_{f3} + x_3 h_{fg3} = 58.04 + (16)(198.60) = 89.34 \text{ kJ/kg}
\]

And, from Table A-9; \( h_4 = 256.86 \text{ kJ/kg} \). Thus,

\[
\dot{m}_{R-22} = (51.11 \text{ kg/min}) \frac{(300.19 - 288.15)}{256.86 - 89.34}
= 3.673 \text{ kg/min}
\]

(b) Consider a control volume enclosing only the refrigerant stream

\[
\hat{Q}_{R-22} = \dot{m}_{R-22} (h_3 - h_4)
\]

\[
= (3.673 \text{ kg/min})(256.86 - 89.34) \text{ kJ/kg}
\]

\[
= 615.3 \text{ kJ/min}
\]

**COMMENT:** For a control volume enclosing only the air stream

\[
\dot{Q}_{air} = \dot{m}_{air} (h_2 - h_1)
\]

\[
= (51.11 \text{ kg/min})(288.15 - 300.19) \text{ kJ/kg}
\]

\[
= -615.3 \text{ kJ/min}
\]

Thus, \( \hat{Q}_{R-22} = -\dot{Q}_{air} \), as expected.
13) A 0.2 m³ tank initially contains R-134a at 8°C and 60% quality. The valve to a supply line is opened and refrigerant at 1 MPa and 120°C is allowed to enter the tank until the pressure reaches 800 kPa, when the valve is closed. At this point, the refrigerant in the tank is saturated vapor. Find:
   a) the final temperature in the tank,
   b) the mass of refrigerant that has entered the tank
   c) the heat transfer between the system and surroundings

\[ Q + \min (h + x \Delta h + \rho_e)_{in} - \min (h + x \Delta h + \rho_e)_{out} = m_u \Delta h_m - m_0 \Delta h_0 \]

\[ Q = m_u \Delta h - m_0 \Delta h_0 - \min \Delta h_m \]

**From Table A+10 @ 8°C:**

\[ \n_1 = 0.0007884 \text{ m}^3/\text{kg} \quad \n_2 = 100.43 \text{ kJ/kg} \]

\[ \n_3 = 0.0525 \text{ m}^3/\text{kg} \quad \n_4 = 231.41 \text{ kJ/kg} \]

\[ \n_5 = \n_1 + x (\n_3 - \n_1) = 0.03182 \text{ m}^3/\text{kg} \]

\[ \n_6 = \n_1 + x (\n_4 - \n_1) = 163.0 \text{ kJ/kg} \]

\[ m_0 = \frac{V_0}{\n_0} = \frac{0.2 \text{ m}^3}{0.03182 \text{ m}^3/\text{kg}} = 6.186 \text{ kg} \]
From Table A-11 @ 800 kPa:

\[ N_e = N_g = 0.255 \text{ m}^3/\text{kg} \quad u_t = u_g = 243.78 \text{ kJ/kg} \]

\[ M_t = \frac{V}{N_e} = \frac{0.2 \text{ m}^3}{0.255 \text{ m}^3/\text{kg}} = 7.843 \text{ kg} \]

(a) \[ T = T_{sat} = 31.33 \degree C \]

\[ M_{in} = M_t - M_0 = 7.843 - 0.286 = 1.557 \text{ kg} \]

From Table A-12 @ 1 MPa, 120 \degree C:

\[ h_{in} = 356.52 \text{ kJ/kg} \]

\[ Q = M_t u_t - M_0 u_0 - M_{in} h_{in} \]

\[ = (7.843 \text{ kg}) \times (243.78 \text{ kJ/kg}) - (0.286 \text{ kg}) \times (143.0 \text{ kJ/kg}) - (1.557 \text{ kg}) \times (356.52 \text{ kJ/kg}) \]

(c) \[ Q = 332.2 \text{ kJ} \]
14) A balloon initially contains 65 m³ of helium gas at atmospheric conditions of 100 kPa and 22°C. Now helium from a large tank at 150 kPa and 25°C is added to the balloon until the pressure in the balloon is 150 kPa. During the filling process the volume varies linearly with pressure. If not heat transfer takes place during the process, find the final temperature in the balloon.

\[ V = \alpha P \]

We can find \( \alpha \) from the initial conditions:

\[ \alpha = \frac{V}{P} = \frac{65 \text{ m}^3}{100 \text{ kPa}} = 0.65 \text{ m}^3/\text{kPa} \]

Energy Balance:

* Note that \( Q = 0 \) but \( W \neq 0 \)

\[ Q + m_i h_{i, in} - W - m_f h_{f, out} = m_f u_2 - m_i u_1 \]

\[ m_i h_{i, in} - W = m_f u_2 - m_i u_1, \]

The volume of the balloon varies linearly with pressure.

\[ V = \alpha P \]
Now let's find the work

\[ W = \int P \, dV = \frac{1}{\alpha} \int V \, dP = \frac{1}{\alpha} \left( \frac{V_b^2 - V_i^2}{2} \right) \]

\[ V_2 = V_i \rho_2 = (0.005 \, m^3/\text{kPa})(150 \, \text{kPa}) = 7.5 \, m^3 \]

\[ W = \left( \frac{1}{0.005 \, m^3/\text{kPa}} \right) \left( \frac{1}{2} \right) (97.5^2 - 6.5^2 \, m^3)^2 \]

\[ W = 4166 \, \text{kPa} \cdot \text{m}^3 = 4166 \, \text{kJ} \]

\[ m_2 = \frac{P_2 V_2}{R T_2} = \frac{(150 \, \text{kPa})(97.5 \, \text{m}^3)}{(2.0769 \, \text{kJ}/\text{kg} \cdot \text{k})(T_2)} \]

\[ m_2 = 7042 \, \text{kg} \]

\[ T_2 \]

We do not have a table for helium but

\[ h_{in} = C_p T_{in} = (5.193 \, \text{kJ}/\text{kg} \cdot \text{k})(298 \, \text{k}) = 1.5475 \, \text{kJ/kg} \]

\[ u_i = C_v T_i = (3.116 \, \text{kJ/kg} \cdot \text{k})(295 \, \text{k}) = 919 \, \text{kJ/kg} \]

Now back to our energy balance

\[ (m_2 - \bar{m}) h_{in} - \bar{W} = m_2 u_2 - \bar{m} \bar{u}_i \]
\[
\left( \frac{7042}{T_2} - 10.61 \right) \text{kg} \left( 1.547 \text{ kJ/kg} \right) - 40.66 \text{ kJ} + \left( 10.61 \text{ kg} \right) \left( 919 \text{ kJ/kg} \right) = \left( \frac{7042}{T_2} \right) \left( 3.116 \text{ kJ} \right) \left( T_2 \right) \text{ K}
\]

This can be solved for \( T_2 \)

\[
T_2 = 334 \text{ K}
\]