1) The average atmospheric pressure in Denver (elevation=1610 m) is 83.4 kPa. Determine the temperature at which water in an uncovered pan will boil in Denver.

\[ p = 83.4 \text{ kPa} = 83.4 \times 10^3 \text{ Pa} \times \frac{1 \text{ bar}}{10^5 \text{ Pa}} = 0.834 \text{ bar} \]

\[ p = 0.834 \text{ bar} \rightarrow T = T_{\text{sat}} = ? \]

From Table A-3:

\[ p = 0.8 \text{ bar} \quad T = 93.50 \degree \text{C} \]
\[ p = 0.9 \text{ bar} \quad T = 96.71 \degree \text{C} \]

\[ \Rightarrow \frac{0.834 - 0.8}{0.9 - 0.8} = \frac{z}{96.71 - 93.5} \]
\[ \Rightarrow z = 1.0914 \]
\[ \Rightarrow T = 93.5 + z = 94.59 \degree \text{C} \]
2) A rigid tank with a volume of 2.5 m$^3$ contains 5 kg of saturated liquid-vapor mixture of water at 75°C. Now the water is slowly heated. Determine the quality $x$ at the initial state. Also, determine the temperature at which the liquid in the tank is completely vaporized.

$v = \text{constant}$

$v_1 = \frac{V}{m} = \frac{2.5 \text{ m}^3}{5 \text{ kg}} = 0.5 \text{ m}^3/\text{kg}$

$T_1 = 75^\circ\text{C}$

From Table A-2 @ $T_1 = 75^\circ\text{C}$:

$v_f = 1.0259 \times 10^{-3} \text{ m}^3/\text{kg}$
$v_g = 4.131 \text{ m}^3/\text{kg}$

$v_1 = v_f + x_1 v_g \Rightarrow x_1 = 0.121$

@ State 2:

$v_2 = v_1 = 0.5 \text{ m}^3/\text{kg}$
$x_2 = 1$

\[
v_2 = v_1 = 0.5 \text{ m}^3/\text{kg} \quad \quad x_2 = 1 \quad \quad v_g = 0.5 \text{ m}^3/\text{kg}
\]

From Table A-2 @ $v_g = 0.5 \text{ m}^3/\text{kg}$:

\[
\frac{0.5 \text{ m}^3/\text{kg} - 0.5089 \text{ m}^3/\text{kg}}{0.3928 \text{ m}^3/\text{kg} - 0.5089 \text{ m}^3/\text{kg}} = \frac{T_2 - 140^\circ\text{C}}{150^\circ\text{C} - 140^\circ\text{C}} \Rightarrow T_2 = 140.7^\circ\text{C}
\]
3) Consider a two phase mixture at 100°C with $x_3 = 0.9$. Determine the specific volume.

From Table A-2 @100°C:

$\nu_f = 1.0435 \times 10^{-3} \text{ m}^3 \text{/kg}$

$\nu_g = 1.673 \text{ m}^3 \text{/kg}$

$$
\nu_3 = \nu_f + x(\nu_g - \nu_f)
$$

$$
\Rightarrow \nu_3 = 1.0435 \times 10^{-3} \text{ m}^3 \text{/kg} + 0.9(1.673 - 1.0435 \times 10^{-3}) \text{ m}^3 \text{/kg}
$$

$$
\Rightarrow \nu_3 = 1.506 \text{ m}^3 \text{/kg}
$$

Note: Once the quality $x$, is known, it can be applied to calculate $\nu$, $h$ or $s$ in the same manner as above.
4) Refrigerant 134a with a quality of 0.4 and a temperature of 12°C is contained in a rigid tank that has a volume of 0.17 m$^3$. Find the mass of liquid present.

\[ \begin{align*}
T_1 &= 12^\circ \text{C} \\
\chi_1 &= 0.4 \\
V &= 0.17 \text{ m}^3
\end{align*} \]

\[\begin{align*}
\text{liquid} & \quad \text{vapor}
\end{align*}\]

\[\begin{align*}
m &= \frac{V}{v}, \quad m_g = x_m, \quad m_f = m - m_g \\
v &= v_f + x v_{fg}
\end{align*}\]

From Table A-10 @ 12°C:

\[\begin{align*}
v_f &= 0.000797 \text{ m}^3/\text{kg} \\
v_g &= 0.046 \text{ m}^3/\text{kg}
\end{align*}\]

\[\begin{align*}
v_1 &= v_f + x v_{fg} \implies v_1 &= 0.000797 \text{ m}^3/\text{kg} + 0.4(0.046 - 0.000797) \text{ m}^3/\text{kg} \\
\implies v_1 &= 0.0189 \text{ m}^3/\text{kg}
\end{align*}\]

Total mass \( m = \frac{V}{v_1} = \frac{0.17 \text{ m}^3}{0.0189 \text{ m}^3/\text{kg}} = 9 \text{ kg} \)

\[\begin{align*}
m_g &= x_m = (0.4)(9) = 3.6 \text{ kg}
\end{align*}\]

\[\begin{align*}
m_f &= m - m_g = 9 \text{ kg} - 3.6 \text{ kg} = 5.4 \text{ kg}
\end{align*}\]
5) Consider Refrigerant-22 at 12°C. It is specific internal energy 144.58 kJ/kg. Determine phase description and enthalpy.

From Table A-7 @12°C:

\[ u_f = 58.77 \text{ kJ/kg} \]
\[ u_g = 230.38 \text{ kJ/kg} \]

So \( u_f < u < u_g \) \( \therefore \) saturated liquid vapor mixture.

\[ x = \frac{(u - u_f)}{(u_g - u_f)} = \frac{(144.58 - 58.77) \text{ kJ/kg}}{(230.38 - 58.77) \text{ kJ/kg}} = 0.5 \]

\[ h = h_f + x h_g = 59.35 \text{ kJ/kg} + 0.5(253.99 - 59.35) \text{ kJ/kg} \]
\[ \Rightarrow h = 156.67 \text{ kJ/kg} \]
6) Determine the temperature of water at a state of \( p = 0.5 \) MPa and \( h = 2890 \) kJ/kg.

\[
P = 0.5 \text{ MPa} = 500 \text{ kPa} = 5 \text{bar}
\]

\[
h = 2890 \text{ kJ/kg}
\]

From Table A-3 for \( p = 5 \) bar

<table>
<thead>
<tr>
<th>p (bar)</th>
<th>( h_f ) (kJ/kg)</th>
<th>( h_g ) (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>640.23</td>
<td>2748.7</td>
</tr>
</tbody>
</table>

\( h > h_g \) so superheated vapor

From Table A-4 @ \( p = 5 \) bar:

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>h (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>2855.4</td>
</tr>
<tr>
<td>240</td>
<td>2939.9</td>
</tr>
</tbody>
</table>

\[\Rightarrow \frac{2890 - 2855.4}{2939.9 - 2855.4} = \frac{z}{240 - 200}\]

\[\Rightarrow z = 16.37\]

\[\Rightarrow T = 200 + z = 216.38 \text{ °C}\]
7) Determine the missing properties and the phase descriptions in the following table for water:

<table>
<thead>
<tr>
<th></th>
<th>T, °C</th>
<th>p, kPa</th>
<th>h, kJ/kg</th>
<th>x</th>
<th>Phase description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>200</td>
<td>200</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>140</td>
<td>1800</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>950</td>
<td>0.0</td>
<td></td>
<td></td>
<td>saturated liquid</td>
</tr>
<tr>
<td>(d)</td>
<td>80</td>
<td>500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(e)</td>
<td>800</td>
<td>3161.7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a)

\[ p = 200 \text{kPa} = 2 \text{bar} \]

\[ x = 0.7 \]

\[ T = T_{\text{sat}} = 120.2 ^\circ \text{C} \]

\[ h = h_f + x h_{fg} = 504.7 + 0.7(2201.9) \Rightarrow h = 2046 \text{kJ/kg} \Rightarrow \text{saturated mixture} \]

(b)

\[ T = 140 ^\circ \text{C} \]

\[ p = 3.613 \text{bar} = 361.3 \text{kPa} \]

\[ h = 1800 \text{kJ/kg} \]

\[ h = h_f + x h_{fg} \Rightarrow 1800 \text{kJ/kg} = 589.13 \text{kJ/kg} + x(2144.7 \text{kJ/kg}) \]

\[ \Rightarrow x = 0.564 \]

\[ \Rightarrow \text{saturated mixture} \]

(c)

\[ p = 950 \text{kPa} = 9.5 \text{bar} \]

\[ x = 0 \]

\[ \text{saturated liquid} \]

\[ p = 950 \text{kPa} = 0.95 \text{MPa} = 9.5 \text{bar} \Rightarrow T = 177.65 ^\circ \text{C} \]

\[ h = h_f = 752.82 \text{kJ/kg} \]

(d)

\[ p = 500 \text{kPa} = 0.5 \text{MPa} = 5 \text{bar} \]

\[ T = 80 ^\circ \text{C} \]

\[ p = 5 \text{bar} \Rightarrow T_{\text{sat}} = 151.9 ^\circ \text{C} \]

\[ T < T_{\text{sat}} \Rightarrow \text{compressed liquid} \]
\[ h \equiv h_{l @ T=80^\circ C} = 334.91 \text{kJ/kg} \]

(e)

\[
\begin{align*}
\text{p} &= 800 \text{kPa} = 0.8 \text{MPa} = 8 \text{bar} \\
\text{h} &= 3161.7 \text{kJ/kg}
\end{align*}
\]

\[
\begin{align*}
\text{p} &= 8 \text{bar} \Rightarrow h_f = 721.11 \text{kJ/kg}, h_g = 2769.1 \text{kJ/kg} \\
h &> h_g \\
\Rightarrow \text{superheated vapor}
\end{align*}
\]

\[
\begin{align*}
\text{p} &= 8 \text{bar} \\
\text{h} &= 3161.7 \text{kJ/kg}
\end{align*}
\]
8) A frictionless piston-cylinder device initially contains 200 L of saturated liquid refrigerant R-134a. The piston is free to move, and its mass is such that it maintains a pressure of 800 kPa on the refrigerant. The refrigerant is now heated until its temperature rises to 50 °C. Calculate the work done during this process.

Refrigerant-134a in a cylinder is heated at constant pressure until its temperature rises to a specified value. The boundary work done during this process is to be determined.

**Assumptions** The process is quasi-equilibrium.

**Properties** Noting that the pressure remains constant during this process, the specific volumes at the initial and the final states are

\[
\begin{align*}
P_1 &= 800 \text{ kPa} \\
& \text{Sat. liquid} \quad v_1 &= v_f @ 800 \text{ kPa} = 0.0008454 \text{ m}^3/\text{kg} \\
P_2 &= 800 \text{ kPa} \\
T_2 &= 50 \degree \text{C} \\
v_2 &= 0.02846 \text{ m}^3/\text{kg}
\end{align*}
\]

**Analysis** The boundary work is determined from its definition to be

\[
m = \frac{V_1}{v_1} = \frac{0.2 \text{ m}^3}{0.0008454 \text{ m}^3/\text{kg}} = 236.6 \text{ kg}
\]

and

\[
W_{b, \text{out}} = \int P \, dv = P(V_2 - V_1) = mP(v_2 - v_1)
\]

\[
= (236.6 \text{ kg})(800 \text{ kPa})(0.02846 - 0.0008454) \text{ m}^3/\text{kg} \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{ m}^3} \right)
\]

\[
= 5227 \text{ kJ}
\]

**Discussion** The positive sign indicates that work is done by the system (work output).
9) A piston-cylinder device contains 50 kg of water at 150 kPa and 25 °C. The cross-sectional area of the piston is 0.1 m². Heat is now transferred to the water, causing part of it to evaporate and expand. When the volume reaches 0.2 m³, the piston reaches a linear spring whose spring constant is 100kN/m. More heat is transferred to the water until the piston rises 20 cm more. Determine (a) the final pressure and temperature and (b) the work done during this process. Also, show the process on p-v diagram.
Water in a cylinder equipped with a spring is heated and evaporated. The vapor expands until it compresses the spring 20 cm. The final pressure and temperature, and the boundary work done are to be determined, and the process is to be shown on a $P-V$ diagram. √

**Assumptions** The process is quasi-equilibrium.

**Analysis** (a) The final pressure is determined from

$$P_3 = P_2 + \frac{F_1}{A} = P_2 + \frac{kx}{A} = (150 \text{kPa}) + \frac{(100 \text{kN/m})(0.2 \text{m})}{0.1 \text{m}^2} = 350 \text{kPa}$$

The specific and total volumes at the three states are

$$T_1 = 25^\circ \text{C}, \quad v_1 \equiv v_f@25^\circ \text{C} = 0.001003 \text{m}^3/\text{kg}$$

$$P_1 = 150 \text{kPa}$$

$$v_1 = m v_1 = (50 \text{kg})(0.001003 \text{m}^3/\text{kg}) = 0.05 \text{m}^3$$

$$V_1 = 0.2 \text{m}^3$$

$$V_3 = V_2 + x_{23} A_p = (0.2 \text{m}^3) + (0.2 \text{m})(0.1 \text{m}^2) = 0.22 \text{m}^3$$

$$v_3 = \frac{V_3}{m} = \frac{0.22 \text{m}^3}{50 \text{kg}} = 0.0044 \text{ m}^3/\text{kg}$$

At 350 kPa, $v_f = 0.0010 \text{ m}^3/\text{kg}$ and $v_g = 0.5243 \text{ m}^3/\text{kg}$. Noting that $v_f < v_i < v_g$, the final state is a saturated mixture and thus the final temperature is

$$T_3 = T_{\text{sat}@350 \text{kPa}} = 138.88^\circ \text{C}$$

(b) The pressure remains constant during process 1-2 and changes linearly (a straight line) during process 2-3. Then the boundary work during this process is simply the total area under the process curve,

$$W_{\text{in/out}} = \text{Area} = P_1 (V_2 - V_1) + \frac{P_2 + P_3}{2} (V_3 - V_2)$$

$$= \left( (150 \text{kPa})(0.2 - 0.05) \text{m}^3 + \frac{(150 + 350) \text{kPa}}{2} (0.22 - 0.2) \text{m}^3 \right) \left( \frac{1 \text{kJ}}{1 \text{kJ/m}^3} \right)$$

$$= 27.5 \text{kJ}$$

**Discussion** The positive sign indicates that work is done by the system (work output).
10) The radiator of a steam heating system has a volume of 20 L and is filled with superheated vapor at 300 kPa and 250°C. At this moment both the inlet and exit valves to the radiator are closed. Determine the amount of heat that will be transferred to the room when the steam pressure drops to 100 kPa. Also, show the process on a p-v diagram with respect to saturation lines.

The radiator of a steam heating system is initially filled with superheated steam. The valves are closed, and steam is allowed to cool until the pressure drops to a specified value by transferring heat to the room. The amount of heat transfer is to be determined, and the process is to be shown on a P-V diagram.

**Assumptions** 1 The tank is stationary and thus the kinetic and potential energy changes are zero. 2 There are no work interactions.

**Analysis** We take the radiator as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

\[
\frac{E_{in} - E_{out}}{\text{Net energy transfer by heat, work, and mass}} = \frac{\Delta E_{system}}{\text{Change in internal, kinetic, potential, etc. energies}}
\]

\[
\Delta Q_{out} = \Delta U = m(u_2 - u_1) \quad (\text{since } W = KE - PE = 0)
\]

\[
Q_{out} = m(u_1 - u_2)
\]

Using data from the steam tables (Tables A-4 through A-6), some properties are determined to be

\[
P_1 = 300\text{ kPa} \quad v_1 = 0.7964\text{ m}^3/\text{kg}
\]

\[
T_1 = 250\text{°C} \quad u_1 = 2728.7\text{kJ/kg}
\]

\[
P_2 = 100\text{ kPa} \rightarrow v_f = 0.001043, \quad v_g = 1.6940\text{ m}^3/\text{kg}
\]

\[
u_f = 417.46, \quad u_{fg} = 2088.7\text{kJ/kg}
\]

Noting that \(v_1 = v_2\) and \(v_f < v_2 < v_g\), the mass and the final internal energy becomes

\[
m = \frac{V_1}{v_1} = \frac{0.020\text{ m}^3}{0.7964\text{ m}^3/\text{kg}} = 0.0251\text{ kg}
\]

\[
x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.7964 - 0.001043}{1.6940 - 0.001043} = 0.470
\]

\[
u_2 = u_f + x_2u_{fg} = 417.46 + (0.470 \times 2088.7) = 1399.1\text{ kJ/kg}
\]

Substituting,

\[
Q_{out} = m(u_1 - u_2)
\]

\[
= (0.0251\text{ kg})(2728.7 - 1399.1)\text{ kJ/kg}
\]

\[
= 33.4\text{ kJ}
\]
11) Two kilograms of saturated liquid water at 50 kPa are heated slowly at constant pressure. During the process 5876 kJ of heat are added. Find the final temperature.

\[ Q = 5876 \text{ kJ} \]

\[ m = 2 \text{ kg} \]

State 1:

\[ p_1 = p_2 = 50 \text{ kPa} \]
\[ x_1 = 0 \]
\[ \int h_1 = h_f = 340.5 \text{ kJ/kg} \]

\[ Q - W = U_2 - U_1 \]

\[ W = \int p \, dV = p \left( V_2 - V_1 \right) = p_2 V_2 - p_1 V_1 \]

\[ \therefore Q = W + \left( U_2 - U_1 \right) = p_2 V_2 - p_1 V_1 + U_2 - U_1 = H_2 - H_1 = m \left( h_2 - h_1 \right) \]

\[ \Rightarrow h_2 = \frac{Q}{m} + h_1 = \frac{5876}{2} + 340.5 = 3278 \text{ kJ/kg} \]

State 2:

\[ p_2 = 50 \text{ kPa} = 0.5 \text{ bar} \]
\[ h_2 = 3278 \text{ kJ/kg} \]

\[ @ p_2 = 0.5 \text{ bar} \Rightarrow h_f = 340.5 \text{ kJ/kg}, \quad h_g = 2645.9 \text{ kJ/kg} \]

\[ h_2 > h_g \Rightarrow \text{superheated vapor} \]

From Table A-4 \( T_2 = 400^\circ \text{C} \)
12) A vessel having a volume of 5 m$^3$ contains 0.05 m$^3$ of saturated liquid water and 4.95 m$^3$ of saturated water vapor at 0.1 MPa. Heat is transferred until the vessel is filled with saturated vapor. Determine the heat transfer for this process.

\[ Q_{12} - W_{12} = U_2 - U_1 \]

KE = PE = 0

State 1:

\[ p_1 = 0.1 \text{ MPa} = 1 \text{ bar} \Rightarrow v_{f1} = 0.001043 \text{ m}^3/\text{kg}, \quad v_{g1} = 1.694 \text{ m}^3/\text{kg} \]

\[ u_{f1} = 417.36 \text{ kJ/kg}, \quad u_{g1} = 2506.1 \text{ kJ/kg} \]

\[ m_{f1} = \frac{V_f}{v_{f1}} = \frac{0.05 \text{ m}^3}{0.001043 \text{ m}^3/\text{kg}} = 47.94 \text{ kg} \]

\[ m_{g1} = \frac{V_g}{v_{g1}} = \frac{4.95 \text{ m}^3}{1.694 \text{ m}^3/\text{kg}} = 2.92 \text{ kg} \]

\[ U_1 = (m_f u_f)_1 + (m_g u_g)_1 = (47.94 \text{ kg})(417.36 \text{ kJ/kg}) + (2.92 \text{ kg})(2506.1 \text{ kJ/kg}) = 27326 \text{ kJ} \]

State 2:

\[ m = m_{f1} + m_{g1} = 47.94 \text{ kg} + 2.92 \text{ kg} \]

\[ v_2 = \frac{V}{m} = \frac{5 \text{ m}^3}{50.86 \text{ kg}} = 0.09831 \text{ m}^3/\text{kg} \]

\[ x_2 = 1 \]

\[ v_2 = 0.09831 \text{ m}^3/\text{kg} \]  We need to do interpolation

<table>
<thead>
<tr>
<th>$v_g$ (m$^3$/kg)</th>
<th>$p$ (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09963</td>
<td>20</td>
</tr>
<tr>
<td>0.09831</td>
<td>?</td>
</tr>
<tr>
<td>0.07998</td>
<td>25</td>
</tr>
</tbody>
</table>
\[ p = 20.3 \text{ bar (final pressure)} \]

<table>
<thead>
<tr>
<th>( u_2 ) (kJ/kg)</th>
<th>( p ) (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2600.3</td>
<td>20</td>
</tr>
<tr>
<td>?</td>
<td>20.3</td>
</tr>
<tr>
<td>2603.1</td>
<td>25</td>
</tr>
</tbody>
</table>

\[ u_2 = 2600.5 \text{ kJ/kg} \]

\[ U_2 = m_2u_2 = (50.86 \text{ kg})(2600.5 \text{ kJ/kg}) = 132261 \text{ kJ} \]

\[ Q_{12} = U_2 - U_1 = 104935 \text{ kJ} \]
13) A cylinder fitted with a piston has a volume of 0.1 m$^3$ and contains 0.5 kg of steam at 0.4 MPa. Heat is transferred to the steam until the temperature is 300°C while the pressure remains constant. Determine the work and heat transfer for this process.

\[
W_{12} = \int_1^2 p \, dV = p \int_1^2 dV = p(V_2 - V_1) = pm(v_2 - v_1)
\]

\[
Q_{12} = U_2 - U_1 + W_{12} = m(u_2 - u_1) + mp(v_2 - v_1) = m[(u_2 + p_2v_2) - (u_1 + p_1v_1)]
\]

\[
\Rightarrow Q_{12} = m(h_2 - h_1)
\]

State 1:

\[
v_1 = \frac{V_1}{m} = 0.1 \text{ m}^3/\text{kg} \quad \text{and} \quad p_1 = 0.4 \text{ MPa}
\]

From Table A-3 @ $p_1 = 0.4 \text{ bar}$:

\[
v_f = 0.001084 \text{ m}^3/\text{kg}, \quad v_g = 0.4625 \text{ m}^3/\text{kg}
\]

\[
v_f < v_1 < v_g \Rightarrow \text{liquid} - \text{vapor mixture}
\]

\[
v_1 = v_f + x_1 v_g \Rightarrow 0.2 \text{ m}^3/\text{kg} = 0.001084 \text{ m}^3/\text{kg} + x_1 (0.4625 - 0.001084) \text{ m}^3/\text{kg}
\]

\[
\Rightarrow x_1 = 0.4311
\]

\[
h_1 = h_f + x_1 h_g \Rightarrow h_1 = 604.74 \text{ kJ/kg} + (0.4311)(2738.6 - 604.74) \text{ kJ/kg}
\]

\[
\Rightarrow h_1 = 1524.7 \text{ kJ/kg}
\]

State 2:
\[ p_2 = 0.4 \text{ Mpa} = 4 \text{ bar} \]
\[ T_2 = 300^\circ C \]

\[ p_2 = 0.4 \text{ Mpa} = 4 \text{ bar} \Rightarrow T_{sat} = 143.6^\circ C, \ T > T_{sat} \Rightarrow \text{superheated vapor} \]

Using Table A-4:

\[ p = 3 \text{ bar} \]

<table>
<thead>
<tr>
<th>T (\text{oC})</th>
<th>h (\text{kJ/kg})</th>
</tr>
</thead>
<tbody>
<tr>
<td>280</td>
<td>3028.6</td>
</tr>
<tr>
<td>300</td>
<td>?</td>
</tr>
<tr>
<td>320</td>
<td>3110.1</td>
</tr>
</tbody>
</table>

Using Table A-4:

\[ p = 5 \text{ bar} \]

<table>
<thead>
<tr>
<th>T (\text{oC})</th>
<th>h (\text{kJ/kg})</th>
</tr>
</thead>
<tbody>
<tr>
<td>280</td>
<td>3022.9</td>
</tr>
<tr>
<td>300</td>
<td>?</td>
</tr>
<tr>
<td>320</td>
<td>3105.6</td>
</tr>
</tbody>
</table>
\[
\frac{300 - 280}{320 - 280} = \frac{z}{(3105.6 - 3022.9)} \Rightarrow z = 41.35 \Rightarrow h = 3022.9 + z = 3064.25 \text{kJ/kg}
\]

\[T = 300 \, ^\circ\text{C}\]

<table>
<thead>
<tr>
<th>p (bar)</th>
<th>h (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3069.35</td>
</tr>
<tr>
<td>4</td>
<td>?</td>
</tr>
<tr>
<td>5</td>
<td>3064.25</td>
</tr>
</tbody>
</table>

\[
\frac{4 - 3}{5 - 3} = \frac{z}{(3069.35 - 3064.25)} \Rightarrow z = 2.55 \Rightarrow h_2 = 3064.25 + z = 3066.8 \text{kJ/kg}
\]

We do similar interpolation for \(v_2 = 0.6548 \text{ m}^3/\text{kg}\)

\[
Q_{12} = m(h_2 - h_1) = 0.5 \text{kg}(3066.8 - 1524.7) \text{kJ/kg} = 771.1 \text{kJ}
\]

\[
W_{12} = mp(v_2 - v_1) = (0.5 \text{kg})(400 \text{kPa})(0.6548 - 0.2) \text{m}^3 = 90.96 \text{kJ}
\]
14) A frictionless piston is used to provide a constant pressure of 400 kPa in a cylinder containing steam originally at 200°C with a volume of 2 m³. Calculate the final temperature if 3500 kJ of heat added.

State 1:

\[ p_1 = 400 \text{ kPa} = 4 \times 10^5 \text{ Pa} = 4 \text{ bar} \]
\[ T_1 = 200^\circ \text{C} \]
\[ V_1 = 2 \text{ m}^3 \]

From Table A-3:
\[ p_2 = 4 \text{ bar} \Rightarrow T_{\text{sat}} = 143.6^\circ \text{C}, \quad T > T_{\text{sat}} \Rightarrow \text{superheated vapor} \]

Using Table A-4 for \( p_1 = 4 \text{ bar} \) and \( T_1 = 200^\circ \text{C} \):

\[ v_1 = 0.5342 \text{ m}^3/\text{kg}, \quad u_1 = 2647 \text{ kJ/kg}, \quad h_1 = 2860 \text{ kJ/kg} \]

\[ W_{12} = \int_1^2 p \, dV = p \int_1^2 dV = p(V_2 - V_1) = 400 \text{ kPa}(V_2 - V_1) \]

\[ m = \frac{V_1}{v_1} = \frac{2 \text{ m}^3}{0.5342 \text{ m}^3/\text{kg}} = 3.744 \text{ kg} \]

\[ Q - W = (U_2 - U_1) = m(u_2 - u_1) \]
\[ \Rightarrow 3500 \text{ kJ} - 400 \text{ kPa}(V_2 - V_1) = m(u_2 - u_1) \]
\[ \Rightarrow 3500 \text{ kJ} - 400 \text{ kPa}(3.744v_2 - 2) \text{ m}^3 = (3.744 \text{ kg})(u_2 - 2647) \text{ kJ/kg} \]

This problem requires trial error procedure:
1) Guess a value of \( v_2 \)
2) Calculate \( u_2 \) from above equation
3) If this value checks with \( u_2 \) from steam tables then the guess is correct one.
A) 1) Let \( v_2 = 1 \text{ m}^3/\text{kg} \)
   
   2) \( 3500 \text{kJ} - 400 \text{kPa}[3.744](1) - 2 \text{m}^3 = (3.744 \text{kg})(u_2 - 2647) \text{kJ/kg} \)
   
   \( \Rightarrow u_2 = 3395 \text{kJ/kg} \)

   3) From Table A-4 for:
      
      \[
      \begin{align*}
      p_2 &= 4 \text{ bar} \\
      v_2 &= 1 \text{m}^3/\text{kg} \\
      u_2 &= 3300.2 \text{kJ/kg}
      \end{align*}
      \]

      These two \( u \) values are not the same. So revise value of \( v \).

B) 1) Let \( v_2 = 1.06 \text{ m}^3/\text{kg} \)
   
   2) \( 3500 \text{kJ} - 400 \text{kPa}[3.744](1.06) - 2 \text{m}^3 = (3.744 \text{kg})(u_2 - 2647) \text{kJ/kg} \)
   
   \( \Rightarrow u_2 = 3372 \text{kJ/kg} \)

   3) From Table A-4 for:
      
      \[
      \begin{align*}
      p_2 &= 4 \text{ bar} \\
      v_2 &= 1.06 \text{m}^3/\text{kg} \\
      u_2 &\approx 3372 \text{kJ/kg}
      \end{align*}
      \]

      We now accept that \( u_2 \) values are close enough. So temperature is \( T_2 \approx 640 \text{°C} \).

Second Method:

\[
Q - W = (U_2 - U_1)
\]

\[
W = \int_{1}^{2} p \, dV = \int_{1}^{2} dV = p(V_2 - V_1)
\]

\[
Q = W + (U_2 - U_1) = p(V_2 - V_1) + (U_2 - U_1) \Rightarrow Q = H_2 - H_1 = m(h_2 - h_1)
\]

\[
h_2 = \frac{Q}{m} + h_1 = \frac{3500 \text{kJ}}{3.744 \text{kg}} + 2860 \text{kJ/kg} = 3795 \text{kJ/kg}
\]

From Table A-4 for:

\[
\begin{align*}
   p_2 &= 4 \text{ bar} \\
   h_2 &= 3795 \text{kJ/kg} \\
   T_2 &\approx 641 \text{°C}
\end{align*}
\]
15) Determine the enthalpy change $\Delta h$ of nitrogen, in kJ/kg, as it is heated from 600 to 1000 K, using (a) the empirical specific heat equation as a function of temperature (Table A-21), (b) the $c_p$ value at the average temperature (Table A-20), and (c) the $c_p$ value at room temperature at 300 K.

\[ \frac{\bar{c}_p}{R} = \alpha + \beta T + \gamma T^2 + \delta T^3 \Rightarrow \bar{c}_p = \left(8.314 \text{ kJ/kmol.K}\right)\left[\alpha + \beta T + \gamma T^2 + \delta T^3\right] \]

From Table A-21:
\[ \alpha = 3.675, \quad \beta = -1.208 \times 10^{-3}, \quad \gamma = 2.324 \times 10^{-6}, \quad \delta = -0.632 \times 10^{-9} \]

\[ \Delta \bar{h} = \int_{T_i}^{T_f} \bar{c}_p dT = \left(8.314 \text{ kJ/kmol.K}\right)\left[\alpha T + \frac{1}{2} \beta T^2 + \frac{1}{3} \gamma T^3 + \frac{1}{4} \delta T^4\right]_{T_i}^{T_f} \]

\[ \Rightarrow \Delta \bar{h} = \left(8.314 \text{ kJ/kmol.K}\right)\left\{3.675[1000 – 600] + \frac{1}{2}[-1.208 \times 10^{-3}[1000^2 – 600^2]] + \frac{1}{3}[2.324 \times 10^{-6}[1000^3 – 600^3]] + \frac{1}{4}[-0.632 \times 10^{-9}[1000^4 – 600^4]]\right\} \]

\[ \Delta \bar{h} = 12544 \frac{\text{kJ}}{\text{kmol}} \]

\[ \Delta h = \frac{\Delta \bar{h}}{M} = \frac{12544 \text{ kJ/kmol}}{28.01 \text{ kg/kmol}} = 447.8 \frac{\text{kJ}}{\text{kg}} \]

b) From Table A-20 at $T_{\text{avg}} = 800$ K, $c_{p,\text{avg}} = 1.121 \text{ kJ/kg.K}$

\[ \Delta h = c_{p,\text{avg}}(T_2 – T_1) = 1.121 \frac{\text{kJ}}{\text{kg.K}}(1000 – 600) \text{ K} \Rightarrow \Delta h = 448.4 \frac{\text{kJ}}{\text{kg}} \]

c) Taking the $c_p$ at room temperature from Table A-20:

\[ c_p = 1.039 \text{ kJ/kg.K} \]

\[ \Delta h = c_p(T_2 – T_1) = 1.039 \frac{\text{kJ}}{\text{kg.K}}(1000 – 600) \text{ K} \Rightarrow \Delta h = 415.6 \frac{\text{kJ}}{\text{kg}} \]
16) A rigid tank has a volume of 400 cm$^3$ contains air initially at 22°C and 100 kPa. A paddle wheel stirs the gas until the temperature is 428°C. During the process, 600 J of heat are transferred from air to surroundings. Calculate the work done by the paddlewheel two ways:

(a) assuming constant specific heat
(b) assuming variable specific heat

(a) $c_v =$ constant

$\Delta U = Q - W$

$\Delta U = mc_v(T_2 - T_1)$

$T_{\text{avg}} = \frac{22 + 428}{2} \approx 225 ^\circ \text{C} = 498 \text{ K}$

From Table A-20:

$c_v = 0.742 \text{ kJ/kg.K} = 742 \text{ J/kg.K}$

$pV = mRT \Rightarrow m = \frac{pV}{RT} \Rightarrow m = \frac{pVM}{RT}$

$m = \frac{(28.97 \text{ kg/kmol})(100 \text{ kPa})(400 \text{ cm}^3)(m^3/10^6 \text{ cm}^3)}{(8.314 \text{ kJ/kmol.K})(295 \text{ K})} = 0.0004725 \text{ kg}$

$W = Q - \Delta U = Q - mc_v(T_2 - T_1) = -742.3 \text{ J}$

(b) variable $c_v$ with temperature:

$\Delta U = Q - W$

$\Delta U = m(c_{v_2}T_2 - c_{v_1}T_1) \Rightarrow Q - W = m(c_{v_2}T_2 - c_{v_1}T_1)$

From Table A-20:
At $T_1 = 295\, K$  
$c_{v1} = 0.718\, kJ/kg.K = 718\, J/kg.K$  
$T_2 = 701\, K$  
$c_{v2} = 1.098\, kJ/kg.K = 1098\, J/kg.K$

$pV = mRT \Rightarrow m = \frac{pV}{RT} \Rightarrow m = \frac{pV}{RT}$

$m = \frac{(28.97\, kg/kmol)(100\, kPa)(400\, cm^3)(m^3/10^6\, cm^3)}{(8.314\, kJ/kmol.K)(295\, K)} = 0.0004725\, kg$

$Q - W = U_2 - U_1$

$U_2 = mc_{v2}T_2 \Rightarrow U_2 = (0.0004725\, kg)(1098\, J/kg.K)(701\, K) = 363.682\, J$

$U_1 = mc_{v1}T_1 \Rightarrow U_1 = (0.0004725\, kg)(718\, J/kg.K)(295\, K) = 100.080\, J$

$W = Q - (U_2 - U_1) = -600\, J - (363.682 - 100.080) = 863.602\, J$
17) A mass of 15 kg of air in a piston-cylinder device is heated from 25 to 77°C by passing current through a resistance heater inside the cylinder. The pressure inside the cylinder is held constant at 300 kPa during the process, and a heat loss of 60 kJ occurs. Determine the electric energy supplied, in kWh.

A cylinder is initially filled with air at a specified state. Air is heated electrically at constant pressure, and some heat is lost in the process. The amount of electrical energy supplied is to be determined.

**Assumptions**
1. The cylinder is stationary and thus the kinetic and potential energy changes are zero.
2. Air is an ideal gas with variable specific heats.
3. The thermal energy stored in the cylinder itself and the resistance wires is negligible.
4. The compression or expansion process is quasi-equilibrium.

**Properties**

The initial and final enthalpies of air are

\[ h_1 = h_{298 \text{ K}} = 298.18 \text{ kJ/kg} \]
\[ h_2 = h_{350 \text{ K}} = 350.49 \text{ kJ/kg} \]

**Analysis**

We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this closed system can be expressed as

\[ Q - W = \Delta U \]

\[ W_{e,in} - Q_{out} - W_{e,pass} = \Delta U \]

\[ W_{e,in} = m(h_1 - h_2) + Q_{out} \]

since \( \Delta U + W_e = \Delta H \) during a constant pressure quasi-equilibrium process.

Substituting,

\[ W_{e,in} = (15 \text{ kg})(350.49 - 298.18) \text{kJ/kg} + (60 \text{ kJ}) = 845 \text{ kJ} \]

or,

\[ W_{e,in} = (845 \text{ kJ}) \left( \frac{1 \text{ kWh}}{3600 \text{ kJ}} \right) = 0.235 \text{ kWh} \]

**Alternative solution**

The specific heat of air at the average temperature of \( T_{ave} = \frac{(25 + 77)}{2} \) 51°C = 324 K is, from Table \( C_p,ave = 1.0065 \text{ kJ/kg°C} \). Substituting,

\[ W_{e,in} = mC_p(T_2 - T_1) + Q_{out} \]

\[ = (15 \text{ kg})(1.0065 \text{ kJ/kg°C})(77 - 25) \text{°C} + 60 \text{ kJ} = 845 \text{ kJ} \]

or,

\[ W_{e,in} = (845 \text{ kJ}) \left( \frac{1 \text{ kWh}}{3600 \text{ kJ}} \right) = 0.235 \text{ kWh} \]

**Discussion**

Note that for small temperature differences, both approaches give the same result.
18) Helium (He) gas initially at 2 bar and 200 K undergoes a polytropic process with \( n = k \) to a final pressure of 14 bar. Determine work and heat transfer for the process, each in kJ/kg of helium. Assume ideal gas behavior.

\[ Q - W = \Delta U \]

\[
W = \int_{v_i}^{v_f} p\,dV = m \int_{v_i}^{v_f} v^k \cdot \frac{dV}{v^k} = m \left( \frac{p_2 v_2 - p_1 v_1}{1 - k} \right) = mR \left( T_2 - T_1 \right) \\
\]

To find \( T_2 \) use:

\[
T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} \\
\]

\[
k = \frac{c_p}{c_v} = \frac{2.5}{1.5} = 1.667 \\
\]

\[
\Rightarrow T_2 = 200(14/1) = 2800 K = 575 K \\
\]

\[
\Rightarrow \frac{W}{m} = \frac{(8.314/4.003)(575 - 200)}{1 - 1.667} = -1167.7 \text{ kJ} \\
\]

\[
Q = m(u_2 - u_1) + W \Rightarrow \frac{Q}{m} = u_2 - u_1 + \frac{W}{m} = c_v(T_2 - T_1) + \frac{W}{m} \\
\]

\[
c_p - c_v = R \\
\frac{c_p}{c_v} - 1 = \frac{R}{c_v} \Rightarrow k - 1 = \frac{R}{c_v} \\
\]

\[
\therefore c_v = \frac{R}{k - 1} \\
\Rightarrow \frac{Q}{m} = \frac{R}{k - 1}(T_2 - T_1) + \frac{R(T_2 - T_1)}{1 - k} = 0 \\
\]
19) Determine the specific volume of superheated water vapor at 1.6 MPa and 225 °C, using (a) the ideal-gas equation, (b) the generalized compressibility chart, and (c) the steam tables. Also determine the error involved in the first two cases.

\[ R = \frac{\bar{R}}{M} = \frac{8.314 \text{kJ/kmol.K}}{18.02 \text{kg/kmol}} = 0.4615 \text{kJ/kg.K} \]

\[ T_c = 647.3 \text{K} \]

\[ p_c = 220.9 \text{bar} = 22.09 \text{MPa} \]

a) ideal gas law:

\[ v = \frac{RT}{p} = \frac{(0.4615 \text{kJ/kg.K})(498 \text{K})}{1600 \text{kPa}} = 0.14364 \text{m}^3/\text{kg} \]

b) compressibility chart, see Figure A-1, Figure A-2, Figure A-3:

\[ p_R = \frac{p}{p_c} = \frac{1.6 \text{MPa}}{22.09 \text{MPa}} = 0.072 \]

\[ T_R = \frac{T}{T_c} = \frac{498 \text{K}}{647.3 \text{K}} = 0.769 \]

\[ Z = 0.935 \]

\[ v = Z \left( \frac{RT}{p} \right) = 0.935(0.14364) = 0.13430 \text{m}^3/\text{kg} \]

\[ \text{error} = \frac{|0.1343 - 0.14364|}{0.14364} \times 100 \approx 7\% \]

c) using superheated table:

\[ p = 1.6 \text{MPa} = 16 \text{bar} \]

\[ T = 225 \degree \text{C} \]

\[ v \approx 0.13287 \]
Example (M1-2011):
Two kilograms of Refrigerant 134a is contained in a piston-cylinder assembly undergoes a constant pressure process from initial state where the pressure is 2 bar and the volume is 0.12 m$^3$ to final state where the volume is 0.24 m$^3$. Determine
(a) the work, and
(b) heat transfer for the process.
(c) Show the process on a p-v diagram.

**KNOWN:** Refrigerant 134a undergoes a constant pressure process. The pressure and the initial and final volumes are specified.

**FIND:** Determine the work and heat transfer for the process, and show the process on a p-v diagram.

**SCHEMATIC & GIVEN DATA:**

\[ P_1 = P_2 = 2 \text{ bar} \]

\[ V_1 = 0.12 \text{ m}^3 \]

\[ V_2 = 2.0 \quad V_1 = 0.24 \text{ m}^3 \]

\[ m = 2 \text{ kg} \]

**ASSUMPTIONS:** (1) The refrigerant is a closed system. (2) The pressure is constant. (3) Kinetic and potential energy effects are constant.

**ANALYSIS:** (a) The work is determined using Eq. (2.17)

\[ W = \int_{V_1}^{V_2} pdV = p(V_2 - V_1) \]

\[ = (2 \text{ bar}) (0.24 - 0.12) \text{ m}^3 \left( \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right) \left( \frac{1 \text{ kJ}}{10^3 \text{ N/m}} \right) \]

\[ = 24 \text{ kJ} \]

(b) The heat transfer is found using the energy balance

\[ \Delta KE + \Delta PE + \Delta U = Q - W \]

Using the assumptions

\[ \Delta U = Q - W \]

or, with \[ \Delta U = m(u_2 - u_1) \]

\[ Q = m(u_2 - u_1) + W \]

From Table A-11, with \[ v_1 = V_1 / m = 0.12 / 2 = 0.06 \text{ m}^3 / \text{kg} \text{ and } p = 2 \text{ bar} \]
\[ p_1 = 2 \text{ bar} = 0.20 \text{ MPa} \quad \Rightarrow \quad v_{lf} = 0.7532 \times 10^{-3} \text{ m}^3/\text{kg} \]

\[ v_{lg} = 0.0993 \text{ m}^3/\text{kg} \]

so \( v_{lf} < v_1 < v_{lg} \) fluid is in the saturated liquid-vapor region. Thus,

\[ x_1 = \frac{v_1 - v_{lf}}{v_{lg} - v_{lf}} = \frac{(0.06 - 0.7532 \times 10^{-3})}{(0.0993 - 0.7532 \times 10^{-3})} = 0.6012 \]

\[ u_1 = u_{fl} + x_1 (u_{gl} - u_{fl}) = 36.69 \frac{\text{kJ}}{\text{kg}} + 0.6012(221.43 - 36.69) \frac{\text{kJ}}{\text{kg}} \]

\[ u_1 = 147.76 \frac{\text{kJ}}{\text{kg}} \]

And, from Table A-12, at \( v_2 = 0.12 \text{ m}^3/\text{kg} \) and \( p = 2 \text{ bar} \)

so \( v_2 > v_{lg} \) fluid is in the superheated region. Thus,

\[ u_2 = 255.66 \frac{\text{kJ}}{\text{kg}} \]

and

\[ Q = (2 \text{ kg})(255.66 - 147.76) \frac{\text{kJ}}{\text{kg}} + (24 \text{ kJ}) \]

\[ Q = 239.8 \text{ kJ} \]

(c) Process on a p-v diagram
Air is confined to one side of a rigid container divided by a partition, as shown in Figure below. The other side is initially evacuated. The air is initially at $p_1 = 5$ bar, and $T_1 = 500$ K, and $V_1 = 0.2 \ m^3$. When the partition is removed, the air expands to fill the entire chamber. Measurements show that $V_2 = 2V_1$ and $p_2 = p_1/4$. Assuming the air behaves as an ideal gas, determine (a) the final temperature, in K, and (b) the heat transfer, kJ.

Initially:
- Air
  - $V_1 = 0.2 \ m^3$
  - $p_1 = 5 \ \text{bar}$
  - $T_1 = 500 \ \text{K}$
- Vacuum

Finally:
- $V_2 = 2V_1$
- $p_2 = \frac{1}{4}p_1$

solution
**PROBLEM 3.128**

**Known:** Data are provided for air on one side of a rigid container. The other side of the container is initially evacuated.

**Find:** For the air, determine the final temperature and $Q$.

**Schematic & Given Data:**

- Removable partition
- Initially: $V_1 = 0.2 \text{ m}^3$, $p_1 = 5 \text{ bar}$, $T_1 = 500 \text{ K}$

- Finally: $V_2 = 2V_1$, $p_2 = \frac{1}{2}p_1$

**Fig. P3.128**

**Engineering Model:**

1. The closed system is the region within the container, ignoring the partition.
2. The air is modeled as an ideal gas.
3. There are no external changes in heat or potential energy.
4. $W = 0$.

**Analysis:**

(a) Using the ideal gas model, and using the initial state, $p_1V_1 = nRT_1$,

$$p_2V_2 = nRT_2$$

Thus,

$$T_2 = T_1 \left( \frac{p_2V_2}{p_1V_1} \right) = 500 \text{ K} \left( \frac{\frac{1}{2}}{1} \right)^2 = 250 \text{ K}$$

(b) An energy balance leads to $\Delta U + \Delta KE + \Delta PE = Q - W$, or

$$Q = m \left( u(T_2) - u(T_1) \right)$$

where

$$m = \frac{p_1V_1}{RT_1} = \frac{(3 \times 10^5 \text{ N/m}^2)(0.2 \text{ m}^3)}{(287 \text{ J/kg K})(500 \text{ K})} = 0.7 \text{ kg}$$

So, with data from Table A-21

$$Q = 0.7 \text{ kg} (178.28 - 359.49) \text{ kJ} \text{ kg}^{-1} = -126.8 \text{ kJ}$$

So, $Q = -126.8 \text{ kJ}$.